1st Junior Penchick Online Mathematical Olympiad

Qualifying Stage, Paper B

(Ages 12 and below)

9 May 2026

INSTRUCTIONS AND INFORMATION

1. **There are 20 questions to be answered in 90 minutes, split into two parts.** The highest possible score for this contest is 48 points.
   - *Part I (multiple-choice):* Each question has four choices (A, B, C, and D), and only one of these choices is correct. Submit the best answer to each question. Two points are given for a correct answer, for a total of 24 points for this section.
   - *Part II (short answer):* Submit the best answer to each question. Three points are given for a correct answer, for a total of 24 points for this section.

   For this paper, answers are either **whole numbers** or **fractions** (in the form of common fractions, mixed fractions, or decimals). All answers must be exact and in their simplest form; otherwise, they will be marked as incorrect. For example, if the answer is $\frac{5}{3}$, then the answers $\frac{5}{3}$, $1\frac{2}{3}$, and $1.\overline{6}$ are accepted (for the decimal, the answer is accepted as long as it is indicated that the decimal is repeating, for example $1.66666\ldots$). However, the answer $\frac{10}{6}$ is not accepted because it is not simplified, and the answer 1.67 is not accepted because it is an estimation of the answer.

2. **Always exhibit academic honesty during the contest.** Do not use calculators, protractors, graph paper, search engines, artificial intelligence, and other external applications, resources, or help while answering the contest. Rulers, compasses, and blank or lined paper are allowed. Cheating in any form is strictly prohibited and will be acted upon based on the violation system.

3. Note that diagrams are **not necessarily drawn to scale**.

4. The top 60% of contestants in this paper will be invited to take the **1st JPOMO Final Stage** on 27 June 2026.

Enjoy the contest!

Part I: Multiple-Choice Questions

Each question has four choices (A, B, C, and D), and only one of these choices is correct. Submit the best answer to each question. Two points are given for a correct answer, for a total of 24 points for this section.

1. Let
$$A = \frac{111111}{222223}, \quad B = \frac{222222}{333334}, \quad C = \frac{444444}{666667}.$$
Which of the following correctly orders them from smallest to largest?

(a) $A < B < C$ (b) $C < B < A$ (c) $B < A < C$ (d) $A < C < B$

2. Renren watches four equally long episodes in a row without taking breaks, finishing all episodes within the day. He watches the first episode at 1:30 pm and finishes the third at 2:50 pm. When does Renren finish watching the fourth episode?

(a) 3:30 pm (b) 3:50 pm (c) 4:20 pm (d) 5:30 pm

3. Pence-chick wants to go to Penchick High School from her house, but apparently her mother Penny-chick wants her to go to the supermarket to buy groceries first and return back. Suppose that she can ride one out of any 3 buses for Pence-chick to go from her house to the supermarket and vice versa, and 4 buses to get from the supermarket to Penchick High School and vice versa, but there is no direct bus from Pence-chick's house to Penchick High School, how many ways can she make her journey with buses while fulfilling her mother's request?

(a) 12 (b) 36 (c) 108 (d) 144

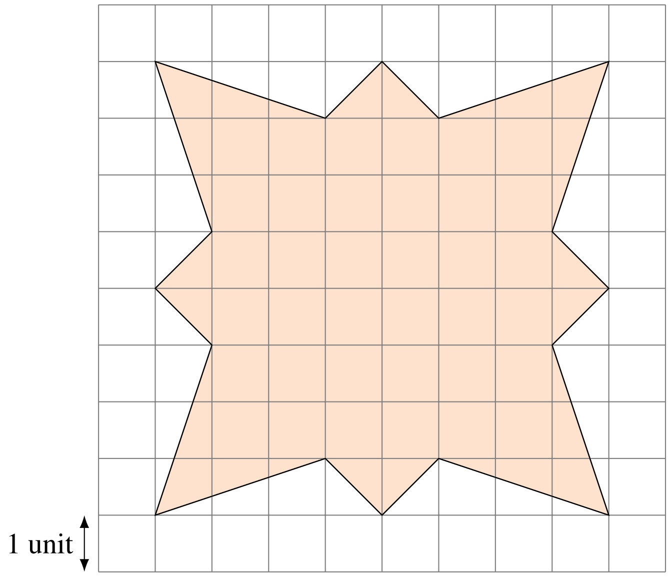
4. Chubby draws three different rectangles on a giant whiteboard. Each rectangle has side lengths that are whole numbers. If each rectangle has an area of 81 square units, then what is the sum of the perimeters of all three rectangles?

(a) 164 (b) 224 (c) 260 (d) 320

5. Penchick has a bag of marbles containing red, blue, and green marbles in the ratio $2:3:5$. After 8 blue marbles are removed, the ratio of red marbles to all marbles becomes $1:4$. How many green marbles were originally in the bag?

(a) 15 (b) 16 (c) 20 (d) 25

6. Given the side length of each square on the grid below is 1 unit, find the area of the shaded region.



(a) 40 (b) 44 (c) 48 (d) 52

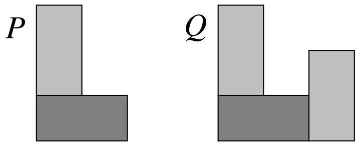
7. Renren flips a fair coin five times. Which event is most likely to happen?

(a) Getting exactly two heads
(b) Getting at least two heads
(c) Getting more heads than tails
(d) Getting more tails than heads

8. Matt, Caitlin, and Elijah are standing in a circle. Michael tells the three of them that he has four hats, two of which are blue and the other two of which are red, and he places three of them on each of Matt's, Caitlin's, and Elijah's heads. The three of them cannot see the hat on their own head nor the unused hat, but they can see the hats of the other two people. Michael asks Matt what color his hat is, to which Matt responds, "Blue!" Caitlin then says, "I know the color of my hat now!" How many of the three used hats are red?

(a) 0 (b) 1 (c) 2 (d) 3

9. Shapes P and Q are formed from two and three identical rectangles, respectively. Their perimeters are 67 centimeters and 88 centimeters respectively. What is the perimeter of one of the rectangles in centimeters?



(a) 42 (b) 44 (c) 46 (d) 48

10. How many positive integers n with $1 \leq n \leq 100$ where n^n is a perfect square?

(a) 40 (b) 45 (c) 50 (d) 55

11. Quadrilateral $RICK$ contains two opposite right angles $\angle RIC$ and $\angle RKC$. If $RC = 25$, $RI = 20$, and $CK = 7$. Find the perimeter of quadrilateral $RICK$.

(a) 59 (b) 66 (c) 67 (d) 90

12. Renren's, Chubby's, and Penrick's favorite numbers are all positive integers. Penrick's favorite number is 41. The last two digits of the product of Renren's number and Penrick's number are 47. The last two digits of the product of Penrick's number and Chubby's number are 53. The last two digits of the product of Renren's number and Chubby's number are 11. What are the last two digits of the product of all three favorite numbers?

(a) 37 (b) 43 (c) 51 (d) 65

Part II: Short-Answer Questions

Submit the best answer to each question. Three points are given for a correct answer, for a total of 24 points for this section.
All answers must be exact and in their simplest form; see the front page of this questionnaire for more details.

1. Today, 9 May 2026, is a Saturday. A fair 100-sided die numbered from 1 to 100 is rolled once, where n is the rolled number. What is the probability that the date n days after May 9, 2026 falls on a weekend (Saturday or Sunday)?

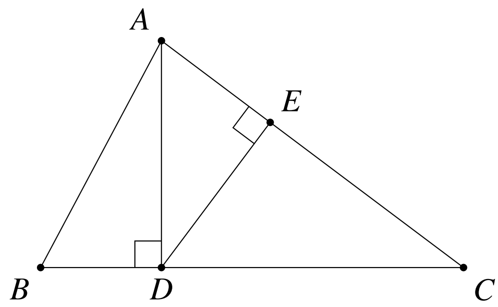
2. An needs help with her math assignment! The numbers a, b, and c are written down by Kohane and An wants to find the least common multiple (LCM) of the numbers. Kohane told her that the three possible pairwise LCMs that can be produced from a, b, and c are 420, 672, and 480, respectively. What number should An write down as the LCM of a, b, and c?

3. Blizzy has 7 different badges that he wants to display in a row. Among them are special a gold badge, a silver badge, and a bronze badge. Blizzy does not want the gold badge to be in between non-special badges. In how many different ways can Blizzy arrange the badges?

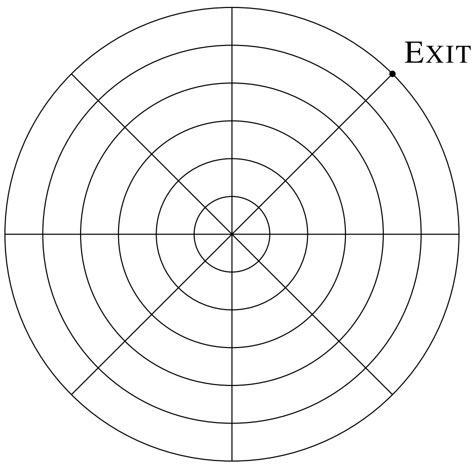
4. A special building has floors numbered from 1 to 99. An elevator in the building has buttons with all two-digit prime numbers, as well as an UP button and a DOWN button. When a prime number p is pressed and the UP button is pressed, the elevator moves up p floors. When a prime number p is pressed and the DOWN button is pressed, the elevator moves down p floors. The elevator starts at floor 1. The elevator is not allowed to go below floor 1 or above floor 99. What is the minimum number of button presses needed to reach floor 99?

5. Tam, Lam, and Sam are working in a laboratory to conduct an experiment. If Tam and Lam can prepare the materials needed for the experiment in 6 hours, Sam and Lam being able to prepare in 5 hours, while all three of them combined can prepare in 4 hours, how many hours does it take for Tam to prepare for the experiment by himself?

6. Let ABC be a triangle with $AB = 17$ and $AC = 25$. Let D be on $\overline{BC}$ such that $\overline{AD}$ is perpendicular to $\overline{BC}$, and let E be on $\overline{AC}$ such that $\overline{DE}$ is perpendicular to $\overline{AC}$. If $AD = 15$, find the length of $\overline{AE}$.



7. Oh no! Penrick has trapped Delta in the maze shown below! Delta is currently at the center of the six circles and must go to the point marked EXIT to exit the maze. She likes to explore, so she can possibly take detours by going around any of the circles. If Delta can only move in the gridlines, outward (away from the center) or around any of the circles without revisiting any point, then she can exit the maze in N ways. How many prime factors does N have?



8. Let A, B, D, E, H, R, Y be distinct nonzero digits such that

$$\overline{BREAD} + \overline{HEAD} = \overline{READY},$$

where $\overline{BREAD}$, $\overline{HEAD}$, and $\overline{READY}$ have five, four, and five digits, respectively. It is guaranteed that only one solution exists. Calculate $B + R + E + A + D$.

Answer Key

Part I: Multiple-Choice Questions

1. a

2. a

3. c

4. c

5. c

6. c

7. b

8. c

9. c

10. d

11. b

12. c

Part II: Short-Answer Questions

1. $\frac{29}{100}$

2. 3360

3. 3600

4. 4

5. 20

6. 9

7. 2

8. 28

Solutions

Some solutions contain follow-up questions. We highly encourage you to answer them to learn more things related to the problem and practice your skills!

Part I: Multiple-Choice Questions

1. Notice that all the numerators are factors of 444444. So, we can rewrite each fraction so that the numerators are all 444444:

$$A = \frac{111111}{222223} \times \frac{4}{4} = \frac{444444}{888892},$$
$$B = \frac{222222}{333334} \times \frac{2}{2} = \frac{444444}{666668},$$
$$C = \frac{444444}{666667}.$$

   We now just have to compare the denominators. We know that the larger the denominator, the smaller the fraction gets, and since $888892 > 666668 > 666667$, we have $\boxed{\text{(a) } A < B < C}$.

2. In total, the first three episodes were $2:50 - 1:30 = 120$ minutes long. Each episode is then $120/3 = 40$ minutes. Thus, Renren watches the fourth episode at $2:50 + 40$ minutes $=$ $\boxed{\text{(a) 3:30 pm}}$.

3. Pence-chick has 3 buses for her to go to the supermarket, and can reuse any of those 3 buses to go back home for her mother. Pence-chick then has 3 buses to go to the supermarket again, and then another 4 buses at her transfer point. Therefore, the total

number of ways Pence-chick can ride buses to Penchick High School while fulfilling her mother's request is $3 \times 3 \times 3 \times 4 = \boxed{\text{(c) } 108}$.

Follow-up question: The neighborhood has built 5 new direct routes from the train station near Pence-chick's house (so she can walk to the train station) to Penchick High School. With these new routes, how many ways can Pence-chick go to Penchick High School while still fulfilling her mother's request?

4. Notice that since all the rectangles are different, they must have different side lengths. Also, let the side lengths of each rectangle be a and b units. The area of each rectangle in units must therefore be $ab = 81$, which means that we have three possible cases for the side lengths of the rectangles:

1. 1 and 81, where the rectangle has a perimeter of $2 \times (81+1) = 164$ units,
2. 3 and 27, where the rectangle has a perimeter of $2 \times (3+27) = 60$ units, and
3. 9 and 9, where the rectangle (which is a square!) has a perimeter of $2 \times (9+9) = 36$ units.

Summing all these up, we have a total sum of $164 + 60 + 36 = \boxed{\text{(c) } 260}$ units.

5. Since the ratio of red to blue to green marbles is $2 : 3 : 5$, we let the number of red marbles be $2x$, the number of blue marbles be $3x$, and the number of green marbles be $5x$. This would make the total number of marbles $2x + 3x + 5x = 10x$.

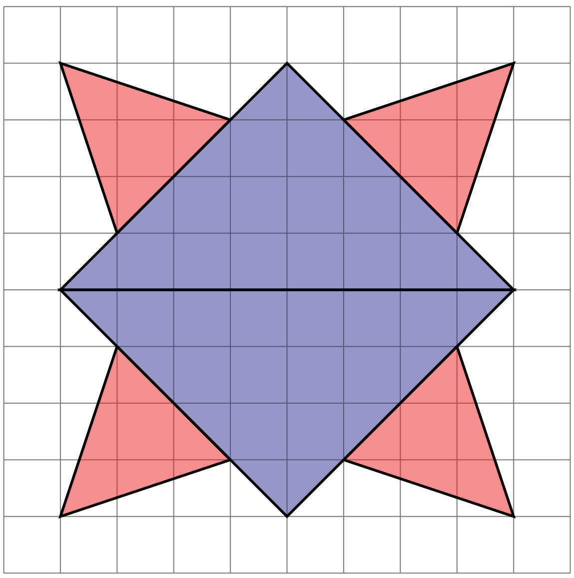
After 8 blue marbles are removed, the total number of marbles would become $10x - 8$. Since the ratio of red marbles to all marbles is $1 : 4$ at this point, we have

$$\frac{2x}{10x-8} = \frac{1}{4}.$$

Cross-multiplying gives $4(2x) = 10x - 8$, or $8x = 10x - 8$. We can transpose $8x$ and 8 to get $8 = 10x - 8x$, or $8 = 2x$, so $x = 8 \div 2 = 4$.

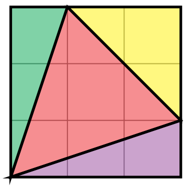
Hence, there are $5x = 5 \times 4 = \boxed{\text{(c) } 20}$ green marbles in the bag.

6. We split the figure as shown, then we find the areas of each part of the figure and add them up to get the total area.



Note that each blue region is a triangle with base 8 and height 4, so they each have an area of $8 \times 4 \div 2 = 16$.

Now we find the area of each red region. We can enclose a red triangle into a 3×3 square as shown, and see that the area of the red triangle is the area of the square with the green, yellow, and purple triangles removed.



- The area of the square is $3 \times 3 = 9$.
- The green triangle has base 1 and height 3, so its area is $1 \times 3 \div 2 = \frac{3}{2}$.
- The purple triangle has base 3 and height 1, so its area is $3 \times 1 \div 2 = \frac{3}{2}$ as well.
- The yellow triangle has base 2 and height 2, so its area is $2 \times 2 \div 2 = 2$.

Therefore, the area of a red triangle is $9 - \frac{3}{2} - \frac{3}{2} - 2 = 4$.

Since there are two blue regions and four red regions, the area of the entire figure is $16 \times 2 + 4 \times 4 = \boxed{\text{(c) } 48}$ square units.

Follow-up question: How else can we solve this problem simply by taking away areas of regions instead of adding areas?

7. Recall that the number of ways we can select k objects from n distinct objects is

$$\binom{n}{k} = \frac{n!}{k!(n-k)!},$$

where

$$n! = 1 \times 2 \times 3 \times \cdots \times n$$

is the product of the positive integers from 1 to n. We will use this formula multiple times throughout the solution. (Sidenote: we read $\binom{n}{k}$ as "n choose k".)

Since each of the events are based on the same action, which is five coin flips, we do not need to compute the actual probabilities, and instead find the number of ways to obtain each event using five coin flips. (This is because the probabilities all share a common denominator, which is $2^5 = 32$. This comes from the fact that there are five coins and each coin can be either heads or tails, giving two options per coin.)

(a) *Getting exactly two heads.* We can choose two of the five coins to be heads. Therefore, there are

$$\binom{5}{2} = \frac{5!}{2!(5-2)!} = 10$$

ways to get two heads.

(b) *Getting at least two heads.* This means that Renren gets two, three, four, or five heads. We already know that there are 10 ways to get two heads. Similarly, there are $\binom{5}{3} = 10$ ways to get three heads, $\binom{5}{4} = 5$ ways to get four heads, and $\binom{5}{5} = 1$ way to get five heads. Thus, there are $10 + 10 + 5 + 1 = 26$ ways to get at least two heads.

(c) *Getting more heads than tails.* This means that Renren gets three, four, or five heads. We already know that there are 10, 5, and 1 ways to do each of these, respectively, so there are $10+5+1=16$ ways to get more heads than tails.

(d) *Getting more tails than heads.* This means that Renren gets three, four, or five tails. There are 10, 5, and 1 ways to do each of these, respectively (figure out why!), so there are also $10+5+1=16$ ways to get more tails than heads.

Since event (b) has the largest number of possible coin flips, which is 26, we know that $\boxed{\text{(b) Getting at most two heads}}$ is the most likely event out of the four.

Follow-up questions:

(a) Find the probabilities for each of the four events.

(b) How can we calculate that there are 26 ways to get at least two heads by finding the number of ways to get **less than** two heads?

(c) If Renren instead flips **six** coins, what is the probability that he gets at least four heads?

Let us now look at some ways we can determine the answer, event (b), faster.

(d) Why can we immediately rule out event (a) from our possible answers?

(e) You may have noticed that events (c) and (d) give the same probabilities. Why is that?

8. Since there are only two red hats and Matt identifies his hat as blue, we know that he sees two red hats. (Caitlin knowing her hat from this information confirms that this is correct.) Thus, Caitlin and Elijah both have a red hat, making exactly $\boxed{\text{(c) 2}}$ people have red hats.

9. Let the perimeter of one of the rectangles be p cm and let the length of one of its shorter sides be x cm.

Before the rectangles in shape P are joined together, their total perimeter is $2p$. However, when they are joined, a length equal to two of the shorter sides is lost. So, the perimeter of shape P is

$$2p-2x=67. \tag{1}$$

Similarly, shape Q consists of three rectangles. Before joining, their total perimeter is $3p$. There are two joins, so a total length equal to four of the shorter sides is lost. So, the perimeter of shape Q is:

$$3p-4x=88. \tag{2}$$

Multiplying equation 1 by 2 gives

$$4p-4x=134.$$

Now subtracting equation 2 gives

$$(4p-4x)-(3p-4x)=134-88.$$

Simplifying both sides gives $p=\boxed{\text{(c) 46}}$ (note that the $4x$'s cancel out on the left side).

10. Note that if n is even, then n^n is a perfect square: since n can be expressed as $2k$ for some positive integer k, we have

$$n^n = n^{2k} = (n^k)^2,$$

which is the square of an integer.

Now, we consider the case where n is odd. Note that if n is a perfect square, then n^n is also a perfect square: since n can be expressed as m^2 for some positive integer m, we have

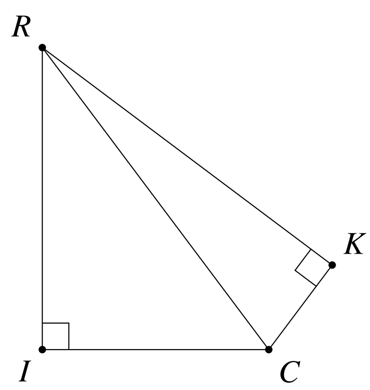
$$n^n - (m^2)^n = m^{2n} = (m^n)^2,$$

which is the square of an integer.

There are $100 \div 2 = 50$ even positive integers less than 100 and 5 odd perfect squares less than 100 (those are 1, 9, 25, 49, and 81), hence there are $50 + 5 = \boxed{\text{(d) } 55}$ possible values of n.

Follow-up question: How many positive integers n less than 200 are there such that n^n is a perfect **cube**?

11. We first draw the diagram, as shown below.



Since $\angle RIC$ is a right angle, we have $RC^2 = RI^2 + CI^2$ by the Pythagorean theorem, or

$$CI = \sqrt{RC^2 - RI^2} = \sqrt{25^2 - 20^2} = 15.$$

Also, since $\angle RKC$ is a right angle, we have $RC^2 = RK^2 + CK^2$ by the Pythagorean theorem, or

$$RK = \sqrt{RC^2 - CK^2} = \sqrt{25^2 - 7^2} = 24.$$

Therefore, the perimeter of quadrilateral $RICK$ is

$$RI + CI + CK + RK = 20 + 15 + 7 + 24 = \boxed{66}.$$

Follow-up questions: The following sets of side lengths form right triangles, and it is important to remember them for math contests:

$$\{3, 4, 5\}, \qquad \{5, 12, 13\}, \qquad \{7, 24, 25\}, \qquad \{8, 15, 17\}.$$

Since all three side lengths are integers, these are called *Pythagorean triples*. If the three side lengths of a Pythagorean triple are relatively prime (meaning their greatest common factor is 1), then they are called *primitive Pythagorean triples*.

(a) Triangle RKC from this question has side lengths 7, 24, and 25, while triangle RIC has side lengths from the first set above with all side lengths multiplied by 5.

Without calculating, do the side lengths $\{14, 48, 50\}$ form a right triangle? Why or why not?

(b) If the side lengths $\{x, y, z\}$ form a right triangle, do the side lengths $\{kx, ky, kz\}$ always form a right triangle for any positive number k? Why or why not?

12. Let Renren's, Penrick's, and Chubby's favorite numbers be R, P, and C, respectively.

We are given the last two digits of each pairwise product:

$$R \times P = 47, \quad P \times C = 53, \quad R \times C = 11.$$

We want the last two digits of $R \times P \times C$.

Consider squaring each pairwise product and multiplying:

$$(R \times P) \cdot (P \times C) \cdot (R \times C) = R^2 \cdot P^2 \cdot C^2.$$

Hence, the square of the last two digits of $R \times P \times C$ must have the same last two digits as $47 \cdot 53 \cdot 11$.

Multiplying only the units digit of $47 \cdot 53 \cdot 11$ gives $7 \cdot 3 \cdot 1 = 21$, so the units digit of $(R \cdot P \cdot C)^2$ is 1.

Now, by checking the 4 options by squaring them, we get:

$$\begin{aligned} 37^2 &= 1369 \implies \text{units digit } 9 \\ 43^2 &= 1849 \implies \text{units digit } 9 \\ 51^2 &= 2601 \implies \text{units digit } 1 \\ 65^2 &= 4225 \implies \text{units digit } 5 \end{aligned}$$

Only 51 squared has units digit 1, matching the units digit of $47 \cdot 53 \cdot 11$. So, only $\boxed{\text{(c) } 51}$ can be correct.

Part II: Short-Answer Questions

1. Since there are 7 days in a week and 9 May 2026 is a Saturday, we know that n must be divisible by 7 in order for the date n days after to fall on a Saturday. Since $98 = 7 \times 14$ is the largest multiple of 7 less than 100, we know that there are 14 possible values of n that can cause the date to fall on a Saturday.

Similarly, we know that n must be one more than a multiple of 7 in order for it to fall on a Sunday. Since $99 = 7 \times 14 + 1$ is the largest number less than 100 that is one more than a multiple of 7, there are $14 + 1 = 15$ possible values of n that can cause the date to fall on a Sunday. (Note that we added 1 because $n = 1$ is also a possibility!)

Therefore, there are $14 + 15 = 29$ possible values of n that can make the date fall on a weekend, and since there are a total of 100 possible values of n (with no restrictions), the probability is $\boxed{\dfrac{29}{100}}$.

2. Since we are dealing with LCMs, the natural first step is to write the prime factorizations of all the numbers:

$$\begin{aligned}\mathrm{LCM}(a,b) &= 420 = 2^2 \times 3 \times 5 \times 7,\\ \mathrm{LCM}(b,c) &= 672 = 2^5 \times 3 \times 7,\\ \mathrm{LCM}(c,a) &= 480 = 2^5 \times 3 \times 5.\end{aligned}$$

Since these three numbers contain all the factors of exactly two of a, b, and c, we know that the LCM of all three of a, b, and c is just the LCM of these three numbers. Comparing the exponents of the prime factors and getting the highest exponents give

$$\mathrm{LCM}(a,b,c) = 2^5 \times 3 \times 5 \times 7 = \boxed{3360}.$$

3. First, we count the total number of arrangements of the 7 distinct badges:

$$7! = 5040.$$

We now subtract the arrangements in which the gold badge is placed between two non-special badges.

Choose two of the four non-special badges to be placed on the left and right of the gold badge. Since order matters, this can be done in 4×3 ways.

Treating these three badges as a fixed block, together with the remaining 4 distinct badges, we now have 5 objects to arrange. They can be arranged in $5!$ ways.

Hence, the number of forbidden arrangements is

$$4 \times 3 \times 5! = 1440.$$

Therefore, the number of valid arrangements is

$$5040 - 1440 = \boxed{3600}.$$

4. The elevator needs to go up a total of $99 - 1 = 98$ floors. We want to minimize the number of button presses, so we need to minimize the number of primes we use.

Recall that the largest two-digit prime number is 97. But, $98 = 97 + 1$, and going up by one floor requires the elevator to go up $n > 1$ floors then go down $n - 1$ floors, which is not allowed because the elevator cannot go above floor 99.

We then look at the next prime, which is 89. We see that $98 = 89 + 9 = 89 + 7 + 2$; this would require a total of $3 \times 2 = 6$ button presses.

We continue listing possible sequences of prime numbers and getting the number of button presses from there.

Prime p	Equation with p	No. of button presses
89	$98 = 89 + 7 + 2$	6
83	$98 = 83 + 13 + 2$	6
79	$98 = 79 + 19$	4

Notice that since 98 itself is not prime, we cannot go below 4 button presses. Hence, the minimum possible number of button presses needed to reach floor 99 is $\boxed{4}$, given by the sequence 79, UP, 19, UP.

5. Let Tam be able to prepare the experiment in T hours, Lam being able to prepare in L hours and Sam being able to prepare in S hours. We first want to express our given information in terms of our non-given. Notice that Tam can finish $\frac{1}{T}$ of the work in 1 hour, Lam can finish $\frac{1}{L}$ of the work in 1 hour and similarly so for Sam. If Tam and Lam work together, they accomplish $\frac{1}{T}+\frac{1}{L}$ of the work in 1 hour, which means that it takes the reciprocal of that expression (in hours) to fully prepare the experiment. Thus, since it takes 6 hours (which is our given), we must have

$$\frac{1}{T}+\frac{1}{L}=\frac{1}{6}.$$

Similarly, we must also have

$$\frac{1}{S}+\frac{1}{L}=\frac{1}{5}$$

and

$$\frac{1}{T}+\frac{1}{L}+\frac{1}{S}=\frac{1}{4}.$$

Notice that we can substitute $\frac{1}{T}+\frac{1}{L}=\frac{1}{6}$ into $\frac{1}{T}+\frac{1}{L}+\frac{1}{S}=\frac{1}{4}$, which gives us $\frac{1}{S}+\frac{1}{6}=\frac{1}{4}$, and after solving for S we get that $S=12$. Now, we can substitute into $\frac{1}{S}+\frac{1}{L}=\frac{1}{5}$ to get that $\frac{1}{12}+\frac{1}{L}=\frac{1}{5}$, or $\frac{1}{L}=\frac{7}{60}$. We can just directly substitute this into $\frac{1}{T}+\frac{1}{L}=\frac{1}{6}$ to get that

$$\frac{1}{T}+\frac{7}{60}=\frac{1}{6}.$$

This yields $\frac{1}{T}=\frac{1}{20}$ or $T=\boxed{20}$, which is our answer. (Again, all our measurements are in hours, so it produces a consistent result.)

6. Since $\angle ADC$ is a right angle, we have $AC^2=AD^2+CD^2$ by the Pythagorean theorem, or

$$CD=\sqrt{AC^2-AD^2}=\sqrt{25^2-15^2}=20.$$

Now, notice that $\angle AED$ is also a right angle, so the area of $\triangle ACD$ can be expressed as $\frac{1}{2}\times AC\times DE=\frac{1}{2}\times 25\times DE$ (with $\overline{AC}$ as the base and $\overline{DE}$ as the height). The area of $\triangle ACD$ can also be expressed as $\frac{1}{2}\times AD\times CD=\frac{1}{2}\times 15\times 20$ (with $\overline{CD}$ as the base and $\overline{AD}$ as the height). We therefore have

$$\frac{1}{2}\times 15\times 20=\frac{1}{2}\times 25\times DE,$$

hence

$$DE=\frac{\frac{1}{2}\times 15\times 20}{\frac{1}{2}\times 25}=\frac{15\times 20}{25}=12.$$

By the Pythagorean theorem, we have $AD^2=AE^2+ED^2$, so

$$AE=\sqrt{AD^2-ED^2}=\sqrt{15^2-12^2}=\boxed{9}.$$

Follow-up question: Notice that point B was not used at all in our solution. (Sometimes, you will have to watch out for unused information!) What is the length of $\overline{BC}$?

7. Firstly, Delta has 8 ways to enter the first (smallest) circle from the center.

Then, she can go to any of the 8 possible exits to the second circle by either going clockwise or counterclockwise to that exit. Since she cannot go around the circle if she is already at her exit point, she has $2 \times (8-1)+1 = 15$ ways to exit the first circle. Similarly, she has 15 ways to exit the second, third, fourth, and fifth circles. This gives Delta a total of 8×15^5 ways to exit the fifth circle from the center.

Now, notice that $\frac{1}{8}$ of these ways to exit the fifth circle make Delta land exactly on the exit point and $\frac{7}{8}$ make her land on some point on the sixth circle other than the exit point. If she lands on the exit point, she must immediately exit, and if not, she can go either clockwise or counterclockwise to the exit point. So, we have

$$\begin{aligned} N &= \frac{1}{8} \times (8 \times 15^5) \times 1 + \frac{7}{8} \times (8 \times 15^5) \times 2 \\ &= 15^5 + 14 \times 15^5 = 15 \times 15^5 = 15^6. \end{aligned}$$

Since

$$N = 15^5 = (3 \times 5)^6 = 3^6 \times 5^6,$$

we know that N has $\boxed{2}$ prime factors, those being 3 and 5.

8. (Note that in this solution, we write $\overline{\cdots X}$ to indicate that a number has X as its units digit.)

Instead of writing it as a long equation, we stack the addition column by column as follows:

$$\begin{array}{ccccc} B & R & E & A & D \\ + & H & E & A & D \\ \hline R & E & A & D & Y. \end{array}$$

Like addition, we work right to left. We narrow down all possible values of D.

The rightmost column shows $D + D = \overline{\cdots Y}$. Since Y is a nonzero digit, then $D \neq 5$. The next column to the right shows

$$A + A + \text{carry-over} = \overline{\cdots D}$$

where the carry-over is the tens digit from the sum of the rightmost column, $D + D$.

If $D + D \leq 9$, the rightmost column will have no carry over. This means $2A = \overline{\cdots D}$. Thus, D is even for this case. The only possible values of even D where $D + D \leq 9$ are

$$D = 2 \quad \text{or} \quad D = 4.$$

Otherwise, if $D + D \geq 10$, a tens digit of 1 is carried over to the next column and $2A + 1 = \overline{\cdots D}$. Thus, D is odd. The only possible values of odd D for $D + D \geq 10$ are

$$D = 7 \quad \text{or} \quad D = 9.$$

We do casework on each value of D.

Case 1: D = 2
If $D = 2$, then $Y = 4$:

$$\begin{array}{ccccc} B & R & E & A & 2 \\ + & H & E & A & 2 \\ \hline R & E & A & 2 & 4 \end{array}$$

Since $A + A = \overline{\cdots 2}$, then either $2A = 2$ or $2A = 12$, giving us

$$A = 1 \quad \text{or} \quad A = 6.$$

If $A = 1$, then the middle column has $E + E = \overline{\cdots 1}$. Since there were no "carry-overs" from the previous column, and $E + E$ is even, this is impossible. Hence, $A = 6$:

$$\begin{array}{ccccc} B & R & \overset{1}{E} & 6 & 2 \\ + & H & E & 6 & 2 \\ \hline R & E & 6 & 2 & 4 \end{array}$$

Now, the middle column has $E + E + 1 = \overline{\cdots 6}$. Since $E + E + 1$ is odd, this is also impossible. Thus, $D \neq 2$.

Case 2: D = 4
If $D = 4$, then $Y = 8$:

$$\begin{array}{ccccc} B & R & E & A & 4 \\ + & H & E & A & 4 \\ \hline R & E & A & 4 & 8 \end{array}$$

Since $A + A = \overline{\cdots 2}$, then either $2A = 4$ or $2A = 14$, giving us

$$A = 2 \quad \text{or} \quad A = 7.$$

If $A = 2$, then $E + E = \overline{\cdots 2}$. Hence, $E = 1$ or $E = 6$. Since the leftmost column has $B = R$, there must have been a "carry-over" of 1 from the previous column. Thus, $R + H > 10$ and $R + H = \overline{\cdots E}$.

E	$R+H$	Possible $\{R,H\}$	With unused digits only
1	11	$\{2,9\},\{3,8\},\{4,7\},\{5,6\}$	$\{5,6\}$
6	15	$\{6,9\},\{7,8\}$	None

Note $R \neq 5$, since this would give $B = 4$, but 4 is already used in digit D. Also, note $R \neq 6$, since this would give $B = H = 5$ and violate the restriction of distinct digits. Hence, $A \neq 2$.

If $A = 7$, then $E + E + 1 = \overline{\cdots 7}$. Note that $E + E \neq 16$, since this would give $E = 8$, but 8 is already a used digit. Again, $R + H > 10$ and $R + H = \overline{\cdots 3}$, so $R + H = 13$.

$R+H$	Possible $\{R,H\}$	With unused digits only
13	$\{4,9\},\{5,8\},\{6,7\}$	None

For either selection of A and E, there will be no valid (R,H). Thus, $D \neq 4$.

Case 3: D = 7
If $D = 7$, then $Y = 4$:

$$\begin{array}{ccccc} B & R & E & \overset{1}{A} & 7 \\ + & H & E & A & 7 \\ \hline R & E & A & 7 & 4 \end{array}$$

Since $A+A+1=\overline{\cdots 2}$, then either

$$A=3 \quad \text{or} \quad A=8.$$

If $A=3$, then the middle column has $E+E=\overline{\cdots 3}$. Since there were no "carry-overs" from the previous column, and $E+E$ is even, this is impossible. Hence, $A=8$:

$$\begin{array}{ccccc} & & {\scriptstyle 1} & {\scriptstyle 1} & \\ B & R & E & 8 & 7 \\ + & H & E & 8 & 7 \\ \hline R & E & 8 & 7 & 4 \end{array}$$

Now, the middle column has $E+E+1=\overline{\cdots 8}$. Since $E+E+1$ is odd, this is also impossible. Thus, $D \neq 7$.

Case 4: $\mathbf{D=9}$

If $D=9$, then $Y=8$:

$$\begin{array}{ccccc} & & & {\scriptstyle 1} & \\ B & R & E & A & 9 \\ + & H & E & A & 9 \\ \hline R & E & A & 9 & 8 \end{array}$$

Since $A+A+1=\overline{\cdots 9}$ and 9 is a used digit, we have $A=4$. Because $E+E=\overline{\cdots 4}$, we know $E=2$ or $E=7$.

Again, we note $B \neq R$ implies $R+H>10$ for $R+H=\overline{\cdots E}$. Therefore:

E	$R+H$	Possible $\{R,H\}$	With unused digits only
2	12	$\{3,9\},\{4,8\},\{5,7\}$	$\{5,7\}$
7	16	$\{7,9\}$	None

Note $R \neq 5$, since this would give $B=4$, but 4 is already used in digit A. However, if $R=7$ and $H=5$, we obtain $B=6$ and $Y=7$, which meets all the problem's conditions.

$$\begin{array}{ccccc} {\scriptstyle 1} & & & {\scriptstyle 1} & \\ 6 & 7 & 2 & 4 & 9 \\ + & 5 & 2 & 4 & 9 \\ \hline 7 & 2 & 4 & 9 & 8 \end{array}$$

Therefore, $B+R+E+A+D=6+7+2+4+9=\boxed{28}$.